Sub-pixel Calibration
to Enable New Science Investigations
under Kepler 2-Wheel Operations

Michael Shao, Bijan Nemati, Chengxing Zhai

Jet Propulsion Laboratory, California Institute of Technology,

4800 Oak Grove Drive, Pasadena, CA 91109-8099, USA

Sean Carey

California Institute of Technology, 1200 E. California Blvd., Pasadena 91125, USA

Alain Leger

Institut d'Astrophysique Spatiale, Univ. Paris-Sud, F91405 Orsay, France

ABSTRACT: This white paper describes the use of sub-pixel calibration techniques to partially recover the photometric accuracy of Kepler, when the spacecraft is operated in the "2 wheel" mode. Its content is based on a brief study conducted after Kepler's third reaction wheel starting showing signs of increased friction. With only 2 wheels, Kepler has one uncontrolled axis of rotation. Solar pressure will cause the spacecraft to rotate about that uncontrolled axis and the pointing of the spacecraft will drift in an "arc." While the pointing is very poor, knowledge of the pointing from the FGS CCDs is quite good. Our brief study suggests that if we calibrate the CCDs and the PSF across the field of view, we could recover photometric accuracy of approximately 70 ppm. For the CCD calibration we would measure the sub-pixel QE variations of the Kepler flight spare CCDs. For the PSF calibration, we would use dithered images of the 100 deg^2 FOV from archived data. The anticipated 70 ppm photometric accuracy is to be compared with the ~30 ppm short-term accuracy achieved by Kepler under 3-wheel operations, limited by stellar astrophysical noise (instrument noise was ~20 ppm). This level of recovered accuracy would enable several science investigations, most notably finding longer period exoplanets–possible only for a mission lasting longer than 4 years. Moreover, our proposed approach enables other investigations that were previously inaccessible: while Kepler's short term (~1 day) photometric accuracy was 30 ppm, its long term astrometric and photometric accuracy was much (~50X) poorer. This calibration procedure can now enable long term photometric accuracy at the ~100 ppm level, which can be of benefit to astroseismology science.

1   INTRODUCTION

We hope to use precise sub-pixel calibration to retrieve approximately 70 ppm photometry of the bright (12 mag) stars in the Kepler field. The decrease in accuracy from ~30 ppm to ~70 ppm would not permit the detection of 1.0 R_{Earth} exoplanets but would allow discovery of ~1.25 R_{Earth} planets. But more important than the raised minimum detectable size is the discovery of planets with longer periods, which a 4 yr mission just couldn't find. Small planets transits must be observed 3 times. An extended mission from, 4 years to 7 years, would enable discovery of planets with periods longer than 1.3 years and shorter than 2.3 years. For some larger planets, where only 2 transits are needed, extending the mission would enable detection of planets with periods > 2 yr and < 3.5 yr.

For transit measurements, the photometric fluctuations of the star are a source of noise. But for astroseismology, these fluctuations are the *signal*. Because of differential stellar aberration, over a period of months stellar images across the Kepler focal plane will move by about 1 pixel and degrade the photometry to ~1%. With precise calibration of the detector and PSF, it may be possible to make a significant improvement in the long term photometric accuracy of the data *that has already been collected* by Kepler's astroseismology program. Photometry with 1~2e-4 accuracy over a period of up to 4 years for 1000's of stars would offer a wealth of new data in the study of the sunspot cycles of stars (11 yr period in the case of our Sun). For this application, the value of a few months of 2 wheel Kepler operation is not primarily in collecting data, but to get empirical data showing that the 4 years of archival data could be calibrated to 1~2e-4 precision on a time scale of months and perhaps years.

Technique Overview

Kepler is the most accurate photometer every built, on the ground or in space. The instrumental precision of Kepler (~1 day time frame) is ~20 ppm. Stellar astrophysical noise limits its transit photometry to ~30 ppm. It's worth understanding why Kepler's precision is so good. Ground based photometry, (even differential photometry) is limited by the atmosphere, normally to ~1e-3, although some groups have reported precision a few times better. There is wide agreement that at or below 1e-4, an observatory in space is needed. Kepler's telescope produces a defocused image ~5 arcsec in diameter placed on a focal plane with 4 arcsec pixels. The large plate scale was needed to cover the 100 deg^2 field of view with only 1e8 pixels. Kepler's focal plane was a mosaic of 42 CCDs totaling 1e8 pixels. While modern *individual* CCDs now have 1e8 pixels, at the time Kepler was built it was considered a large focal plane.

It is well known that the QE of a CCD is not uniform among pixels at even the 0.1% level, much less at 20 ppm. It is also known that the QE within a pixel is not uniform to 0.1%. So how did Kepler achieve such high (20 ppm) precision? Kepler's precision was a result of extremely stable pointing of the telescope. The Kepler spacecraft kept the telescope pointed to ~3 mas, or 0.0007 of a pixel, on a time scale of 15 min. As long as the stellar image was sitting on the same part of the same pixels for days, the photometric precision was extremely high. If the CCD QE was uniform to 3% and the image was stable 0.0007 pixels one would expect the photometry to be stable to $3\% \times 0.0007 \simeq 20$ ppm.

Astrometry is intimately tied to photometry. The ratio of flux falling on adjacent pixels provides information on the "location" of the star. Hence, astrometry at the 1e-4 of the image diameter is possible if the photometry is precise to < 1e-4. For Kepler, the astrometric motion of the stellar image became a key technique in identifying "blended" objects, eclipsing binary stars that were within the ~10 arcsec box used for photometry. As with photometry, astrometry on a short ~1 day timescale was extremely accurate with Kepler because the images didn't move across the focal plane.

With the loss of the 3rd reaction wheel, Kepler is no longer able to precisely point to 3mas on the sky and hence no longer able to do 20 ppm photometry.

Differential stellar aberration

Stellar aberration is caused by the motion of the spacecraft around the Sun. The gross effect is $v/c \simeq$ 20". But over a 10 deg FOV, the differential stellar aberration would be 3" (that is, $\sin(10°) \cdot 20$"). Since the Kepler pixels are 4" across, differential stellar aberration caused most of the stars in the Kepler field to traverse ~ 1 pixel even when the center of the 10 deg field was stable to 3 mas. As a result, long term photometric accuracy was significantly less accurate, ~1%, and long term (~ few months) astrometry was similarly degraded to a large fraction of an arcsec.

Progress since the launch of Kepler

A great deal of progress has been made in characterizing CCD detectors for precision photometry and astrometry since the launch of Kepler. Our efforts in CCD calibration started with the Space Interferometry Mission (SIM, cancelled in 2010), where CCDs were used to measure fringe position with ~1e-5 λ accuracy. More recently, we demonstrated centroiding diffraction limited images with 1e-5 λ/D accuracy (Shao 2013). The key to both precision photometry and astrometry is (1) precise knowledge of the CCD, the $QE(i,j,x,y)$ where i,j represent the pixel index and x,y the variation of QE within each pixel, and (2) precise knowledge of the point-spread function (PSF), i.e. the stellar image. If we have perfect knowledge of both, we can recover accurate photometry and astrometry regardless of where the image lands on the CCD.

The essence of the concept described in this white paper

The ultra-precise detector calibration that makes photometry and astrometry possible at 1e-5 requires calibration of the actual flight detectors. However, our work with backside illuminated CCDs from E2V showed that within a 'batch', the QE and its variations are similar at the 0.1%. Our simulations that arrive at recovering ~70 ppm photometric precision is based on the assumption that the E2V backside CCDs used in Kepler are as repeatable as the E2V backside CCD's we've been studying. This needs to be verified by testing the several, extant flight spare Kepler CCDs. Kepler has a wide field (10×10 deg) and the shape of the PSF changes across that field. However Kepler was observing 150,000 stars at once and there should be more than enough stars to derive the field-dependent PSF for every 1 deg patch of the sky, using sub-pixel dithered images from archival data. This is combined with the precise knowledge of the pointing of the telescope using data from the FGS CCDs at the corners of the Kepler field. Calibration of the sub-pixel QE of the detectors, sub-pixel calibration of the PSFs across the Kepler field, and averaging of pixilation/PSF uncertainty errors that occur when the stellar images are moved across many pixels are the reasons why it may be possible to recover much of the accuracy of the Kepler mission.

2   SUB-PIXEL CALIBRATION TECHNLOLOGY

In this section we describe the sub-pixel calibration technology that enables our proposed solution for Kepler 2-wheel operations. This technology has been developed over the past few years to enable high precision astrometry and is readily applicable to precision photometry.

CCD Characterization

The response of a CCD, for photometric purposes, can be thought of as the set of functions $Q_{mn}(x,y)$ that describe the photometric response of each pixel in the CCD. The pixel response function determines the recorded counts C_{mn} in response to illumination $I(x,y)$:

$$C_{mn} = \iint dx\,dy\,Q_{mn}(x,y)\,I(x,y) \tag{1}$$

where x, y are in principle not confined to the nominal boundaries of a pixel. This model captures non-ideal response aspects such as the variation of QE within a pixel and detection beyond the boundary ore leakage. The leading order aspects of a pixel's response function are its mean quantum efficiency (QE) and effective location.

Characterization of a CCD at the sub-pixel level is possible by illuminating the focal plane with laser light launched from the tip of a series of fibers. Two fibers illuminating the focal plane create fringes on the pixels. If the fiber ends are attached to a thermally stable glass block, the fringe spacing can be a stable reference that is the basis of metrology of the pixels. The fringes can be made to move across the focal plane by shifting the phase of the light coming from one of the fibers. With these moving fringes we can calibrate the pixel response functions of all the pixels, the lowest orders of which are familiar as the QE and the position of the pixel. By appropriately setting the spacings of the fiber tips at the metrology block, the desired fringe spacings can be created to allow the determination of the various Fourier components of the pixel response functions $Q_{mn}(x,y)$. For Kepler, we will have to calibrate the flight spares available on the ground, and assess the degree to which the CCD pixel response functions have commonality among each other. We would then assume that the flight CCD's are well represented by the samples on the ground and ascribe to those CCD's the calibration results obtained from the spares.

To calibrate the pixel response functions, we use the laser metrology configuration illustrated below in Figure 1. A stabilized laser is used to illuminate the focal plane via various pairs of optical fiber tips installed on a 'metrology block'. Light from two point sources creates fringes on the focal plane. The fringe pattern is made to move over the face of the detector by changing the phase of the light coming from one of the two fibers. This is done using a fiber phase shifter driven with a function generator in a sinusoidal pattern over a peak to peak range slightly more than 2π. A 2 x N fiber switch routes each of shifted and unshifted inputs to any of a set of output fibers as commanded. The result is that a variety of fringe patterns are made possible.
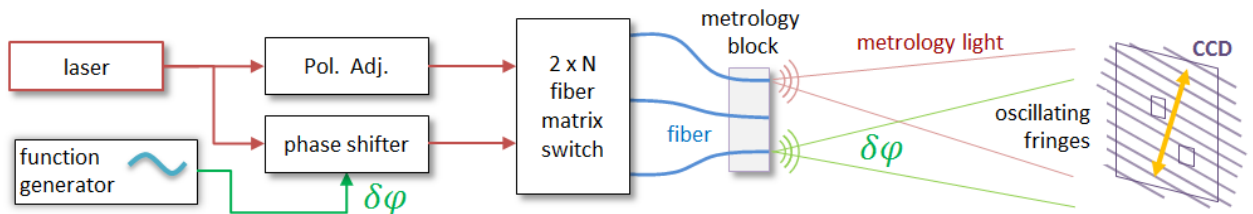

Figure 1: High precision metrology setup for calibrating the focal plane. Laser light is split and one arm is phase shifted. The light is launched from the tips of fibers (one pair in each instance) onto a CCD camera, where moving fringes are observed.

We have already developed this technology in a testbed at JPL geared towards the application of such calibrations towards precision astrometry. The testbed is located inside and at one end of a 40' long vacuum chamber, as seen in Figure 2. At one end is the test CCD (an E2V CCD39). An image from the camera is shown which has both the metrology fringes and 3 of the stars that fit in the field of view of the camera. In normal operation the metrology fringes are turned off during star observations. At the other end is an optical glass bench with an off-axis parabola which images the light from a fiber bundle onto the CCD.
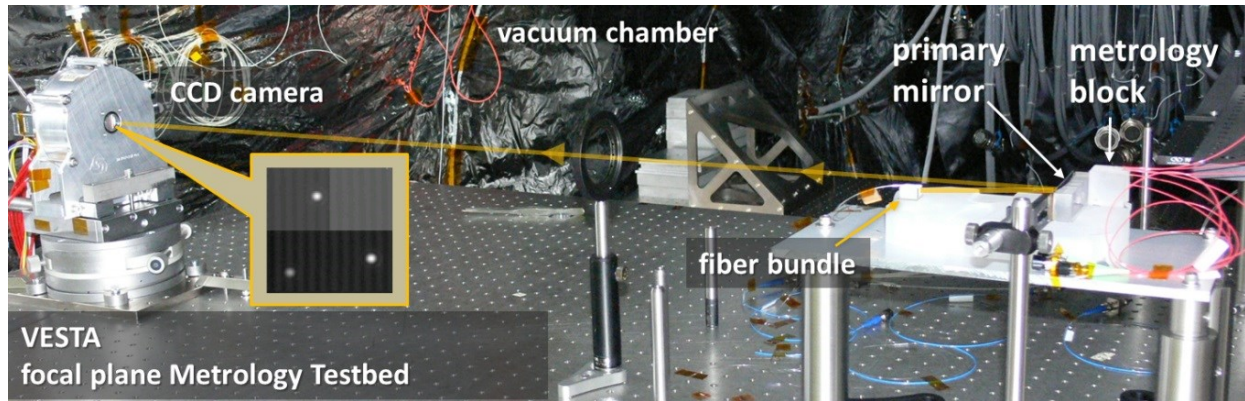
Figure 2: CCD calibration and metrology testbed at JPL.

The fiber switch allows any one or two of the fibers to be activated. For example, the mean QE of each pixel is estimated using single-fiber illumination, which produces a 'flat' field (actually there are gradients due to Gaussian beam, but these can be detrended). For this calibration we use a broadband source (super-luminescent diode). We measured the difference of the measured response between two identical runs. For the tested E2V CCD, the standard deviation of the mean pixel response (QE) differential between two runs was less than 2e-4, implying a per-measurement standard deviation of 1.4e-4 in the fractional QE measurement. This was in fact consistent with the expected photon noise.

As we pointed out earlier, precision astrometry is closely related to precision photometry. CCD characterization enables us to achieve 10^{-5} λ/D level astrometry. Figure 3 shows the results of an astrometry test using the pixel response calibration sketched above. The right side of the figure shows a sample image with three stars. The stellar PSF is oversampled (at about 3 pixels per λ/D). The test consists of displacing the CCD while measuring the distance between pseudo-stars A and B. Without CCD calibration, there is about 1 mpix level of variation in the inter-centroid measured distance when we displace CCD by 1 pixel (24um for the E2V CCD39). Incorporating the pixel calibration data into our centroid estimation, we were able to correct most of the errors and achieve the distance between A and B, varying only at 100 μpix level when moving the CCD by 3 pixels in column. We then group the measurements into three groups corresponding to CCD displacement within intervals [-1.5,-0.5], [-0.5,0.5], [0.5, 1] pixel. The plot shows the result; the standard deviation of the inter-centroid distance between A and B foes down to approximately 10^{-5} λ/D. In pixel units this corresponds to < 5 μpix.
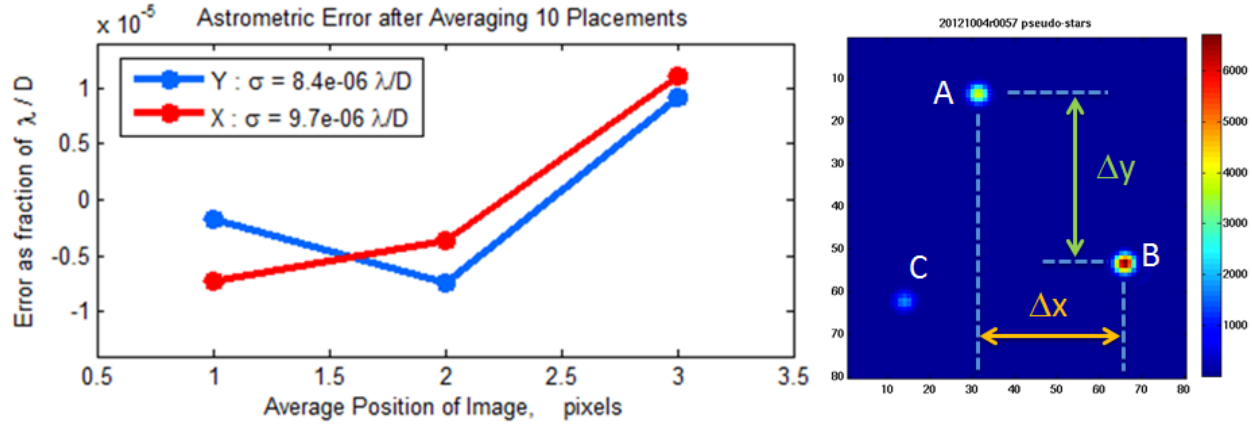
Figure 3: Astrometric test shown that, with pixel calibration, the inter-star centroid distance standard deviation as the CCD is moved 'underneath' the image is about 1e-5 λ/D, or < 5 μpix.

PSF Characterization at the Sub-Pixel Level

To calibrate the photometric errors due to large pointing errors, we must have both knowledge of the PSF and the CCD pixel responses. Kepler CCD pixel size is approximately 4" on the sky, which is much larger than the Nyquist sampling of the diffraction limited PSF (viz. $\lambda/2D \sim 60$ mas). For this reason, Kepler's images are defocused to spread over more than one pixel. According to our simulation results, described below, we would nominally like to have knowledge of PSF with 1/5 pixel resolution so that the error due to deficiency in PSF knowledge is ~50 ppm. To achieve this, we use the dithered images from the Kepler archival data to estimate the PSF at resolution finer than 1 pixel.

Pixelated images represent sampled values on a grid. When the grid spacing (pixel size) is less than the Nyquist frequency of the image (highest frequency content in the image), then, according to the sampling theorem, the image as a continuous function of space can be fully reconstructed, without loss. When the pixels are larger than the Nyquist frequency (i.e. the images are under-sampled), the reconstruction has ambiguities due to frequency aliasing. Thus, it is not possible to reconstruct the Kepler PSF from a single pixelated image as the sampling frequency is much lower than the Nyquist frequency. A common technique to increase sampling frequency is to take multiple images while "dithering" (varying) the location of the image on the CCD with a step size of a fraction of a pixel. Putting multiple images together, the dithered images fill the sampling values between the original CCD pixel grid points. Then the dither step size becomes the effective sampling frequency. There are different ways to combine the dithered images. We can use the parameterization approach of Bryson, et al.[1] Alternatively, we can take Lauer's approach and work in Fourier space.[2] This involves estimating the PSF frequency components using the combined dithered images. The ambiguities due to frequency aliasing are resolved by the dithered images, which are displaced from each other by a factional pixel. Because the dithered image positions are not known precisely, we will simultaneously estimate these positions. Sometimes, the dithered images have a small relative rotation and scale changes among each other.

---

[1] Bryson et al. 2010, ApJL, 713, L97

[2] Lauer, T. R. 1999, "Combining Undersampled Dithered Images," PASP, 111, 227

These rotations and scale changes are solved as well. Due to estimating extra parameters, we will use more than 5x5 dithered images to achieve 1/5 pixel resolution in estimating PSF.

3   OPERATIONS CONCEPT

The operations concept is based on the "default" mode of 2 wheel operation, where the two remaining reaction wheels attempt to stabilize the pointing of the spacecraft (S/C). The rotation of the S/C cannot be controlled around a rotation axis that is the cross product of the rotation axes of the two remaining wheels. Solar radiation pressure will cause the S/C to precess about this axis and the thrusters are used in a one sided "ping" mode to keep the S/C pointed roughly at the Kepler field.

The star images will then follow an arc that is ~1 arcsec wide and ~1 deg long. Pixels along this arc would be transmitted back to Earth. In our brief study a few months ago, we felt that there was sufficient data bandwidth to send back the pixels for ~10% of the stars in the Kepler field. Clearly a more detailed study is needed to maximize the science return within very tight cost constraints. The more flexible or capable the on-board computer is in selecting what pixels to send down, the higher the science return.

4   SIMULATIONS

We conducted a brief simulation study to assess the performance of the proposed approach for recovering the photometry accuracy for 2-wheel operation.

Photometry is determined by the integral of the PSF and the detector responses. We simulated a PSF based on Kepler's optical specifications, e.g. the sizes of the telescope and central obscuration, using Fourier optics. Figure 4 shows the simulated PSF with 1/60 pixel resolution (a) and the measured Kepler PSF interpolated to 1/50 pixel scale (b). Here, we defocused the image to bring the overall spot size close to that of the measured Kepler PSF. We also added 1.5 waves of coma to make the shape similar. Additional random wavefront aberrations of ¼ waves RMS following a $1/f^2$ law were also included to simulate a more realistic situation. Our simulation resolution is 1/60 pixel.
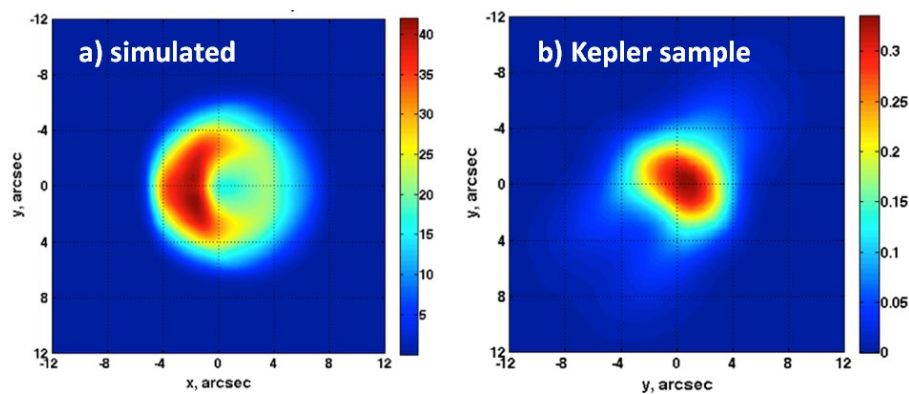

Figure 4, (a) simulated PSF with 1/60 pixel resolution (b) Measured Kepler PSF interpolated to 1/50 pixel resolution

The non-uniformity of CCD pixel responses is the main cause of degraded photometry when the pointing errors become large. We simulate the detector using 60x60 sub-pixels per pixel, i.e. 1/60 pixel resolution. The average detection quantum efficiency (QE) was assumed to be 0.8, with 2% inter-pixel random QE variation introduced as white noises. We also included 5% intra-pixel random QE variations at 1/10 pixel scale as white noises. This yields 10 ppm systematic error for 3 mas pointing rms, a level similar to Kepler's performance. An example of such CCD response map is shown in Figure 5.
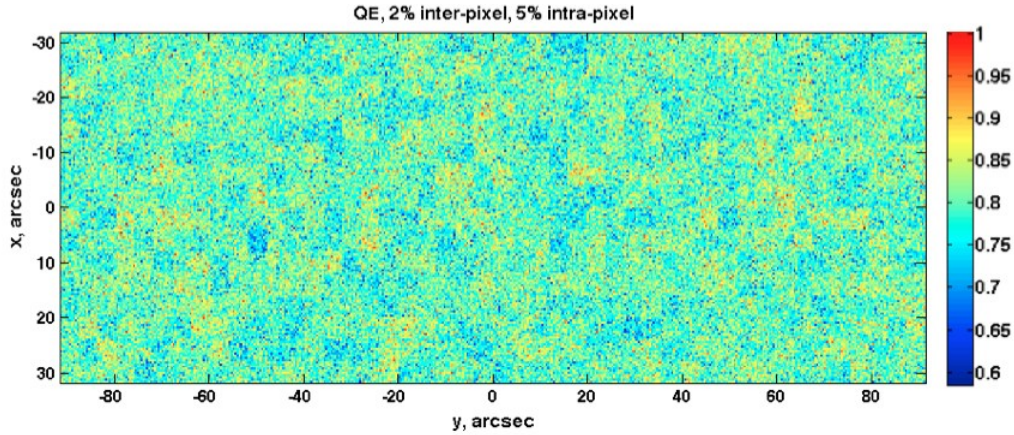
Figure 5: One example of a simulated pixel QE map.

Expected Performance

With only two wheels plus the thrusters, a viable long-term operational scenario inevitably leads to a drift in attitude. Figure 6 shows a simulated streaked image of a star corresponding to 2 arcmin attitude drift during a single exposure. Because the photometry of the star involves all the pixels in the streaked image and the streaked images land on different sets of pixels for different exposures, the non-uniform pixel responses lead to large photometric error.
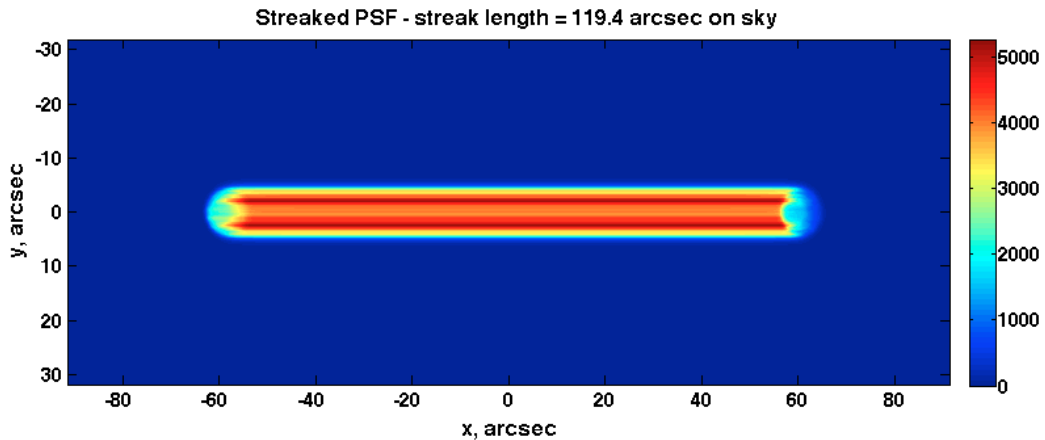

Figure 6: a simulated streaked image of a star.

Figure 7 shows the photometric systematic errors due to 2 arc-minute streaks. Each point corresponds to one realization of randomly generated CCD pixel responses. From 100 random runs, we found that

the RMS of the photometric systematic error is 3e-3; this is 150 times of the nominal Kepler photometric error of 20 ppm.
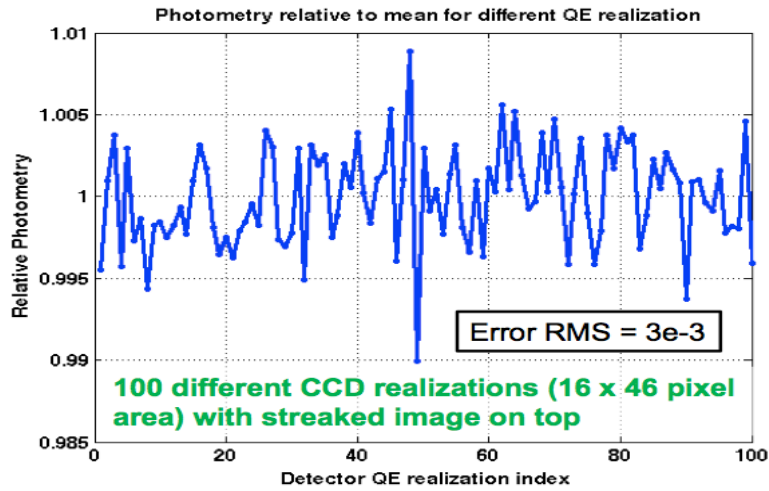

Figure 7: Monte Carlo of the degraded performance due to pointing error with no calibration.

To improve the precision of the photometry, we need to calibrate both the CCD pixel responses and the PSF. It is convenient to divide the calibration errors into knowledge errors of PSF and CCD responses. The main PSF calibration error comes from the resolution, as the Kepler PSF is under-sampled. Figure 8 (a) displays the photometric error RMS due to the finite resolution of the PSF as function of the resolution, assuming the knowledge of QE is perfect. At the level of 1/5 pixel resolution (5 sub-pixels) for PSF knowledge, the photometric error is approximately 52 ppm. We will adopt this resolution as the nominal case. Figure 8 (b) shows the photometric error due to knowledge error of pixel responses assuming perfect knowledge of PSF. The photometric error using a Monte Carlo with 100 realizations (with 0.2% rms pixel QE error) is found to be approximately 54 ppm. The error is directly proportional to the QE knowledge error. The errors due to imprecise PSF knowledge and QE knowledge are independent. The root mean square of the 50 ppm level errors due to PSF knowledge at 1/5 pixel resolution and 0.2% QE calibration error implies that the total error is approximately 70 ppm.
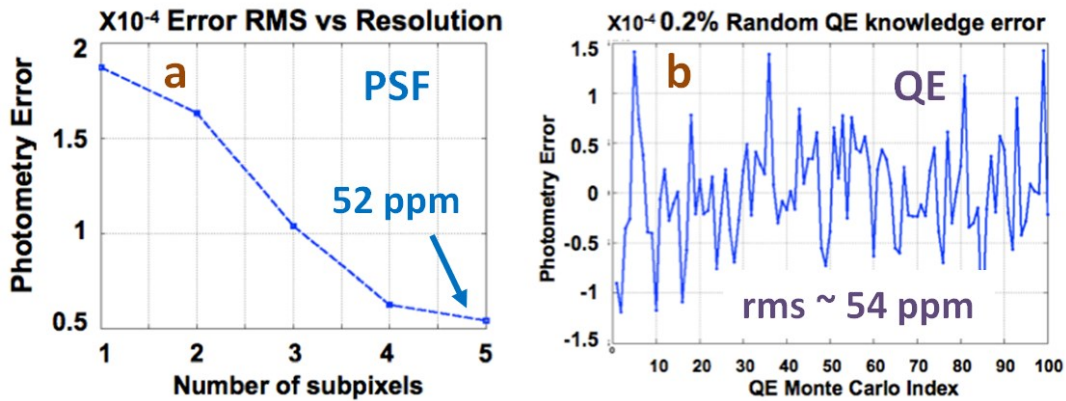

Figure 8: Photometric error RMS as function of resolution of PSF knowledge (a) and photometric error due to 0.2% knowledge Error in QE (b). Each can be expected to contribute about 50 ppm at the recommended level of calibration.

Figure 9 shows of the photometric error from 100 Monte Carlo runs with PSF knowledge at resolution of 1/5 pixel and random 0.2% QE calibration errors. The standard deviation is 72 ppm, which is consistent with our expectation. So, if we measure the PSF with 1/5 pixel resolution and pixel response to precision of 0.2% QE, we improve the photometry error from 3000 ppm to just over 70 ppm, a factor of 40 reduction.
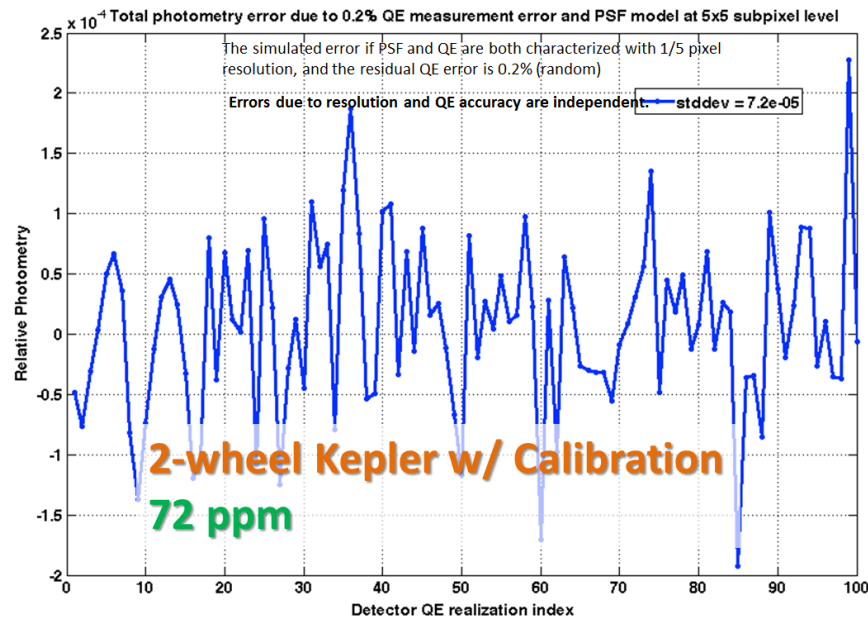

Figure 9: Monte Carlo of the total photometric error assuming 0.2% QE knowledge error and 5x5 sub-pixel resolution of PSF.

5   SUMMARY AND CONCLUSIONS

This white paper describes an approach that can potentially recover most of the photometric precision of Kepler, using precise calibration of the sub-pixel variations in QE, the sub-pixel shape of the PSF across the Kepler FOV, and the averaging of the residual errors that happen when the images are moved across many pixels.

The science goal of this effort would be to continue observing the Kepler targets to find longer period planets, a goal that only a longer mission time line can enable. The targets we would concentrate on are ones that have had one or two observed transits, where a second or third observed transit would confirm the existence of the planet. Other targets of interest would be ones where the cumulative SNR of the prior detections were slightly below 7 sigma and an additional one or two transits would push the SNR over 7.

Kepler data collected over the first 4 years has extremely high photometric precision on the short term scale (days), but much poorer (~1%) precision over time scales of months and years. This is due to differential stellar aberration which causes the star images across the FOV to move by order 1 pixel. The calibration process described here could potentially recover 1~2e-4 long term photometric accuracy for the vast existing Kepler data set.